University of California Santa Cruz Physics Department

Statistical Mechanics 219 SQ 2018

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Homework 3
Solve by 1 May 2018

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1. A homogenenous substance with a positive heat capacity is expanding isobarically. Demonstrate that its entropy is increasing if the thermal expansion coefficient is positive and, conversely, the entropy is decreasing if the thermal expansion coefficient is negative.

Hint: $\left(\frac{\partial X}{\partial Y}\right)_F \equiv \frac{\left(\frac{\partial X}{\partial Z}\right)_F}{\left(\frac{\partial Y}{\partial Z}\right)_F}$

2. Compute the change of the entropy ΔS of a system, whose mass is m, the specific heat is c and the thermal expansion coefficient is zero, when its temperature is increased from T_1 to T_2.

3. Demonstrate that $\left(\frac{\partial T}{\partial V}\right)_S = -\frac{T}{C_V}\left(\frac{\partial P}{\partial T}\right)_V$.

4. The pressure of the electromagnetic radiation of a black body is given by $P = \frac{\rho}{3}$, where $\rho = aT^4$ is its energy density at temperature T; a is a known constant.

(a) Compute the entropy change of this radiation when it is expanding isothermally from volume V_1 to volume V_2.

(b) How much work does it do in a Carnot cycle with a heater and a cooler at temperatures T_1 and T_2?

5. The dependency of the length ℓ of a certain spring on the force f, with which you pull it, has been measured in experiment and has the form $f = A(\ell)T$, where T is the temperature and $A(\ell)$ is some function of ℓ only.

(a) How does the internal energy of this spring depend on its length?

(b) Demonstrate that the entropy of the spring decreases when you pull it isothermally.