UNIVERSITY OF CALIFORNIA SANTA CRUZ PHYSICS DEPARTMENT

Condensed Matter 219 SQ 2018

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Solve by 13 June

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1. The compressibility of a metal is often determined by the pressure of the electron gas in it. Compute the electron pressure P and the compressibility β_T of copper at $T = 0$. The electron concentration in copper is $n = 8.5 \cdot 10^5 cm^{-3}$. Assume that the electron dispersion is quadratic and isotropic and that the effective mass m^* matches the bare electron mass m_e.

To solve this problem, you may accomplish several steps:

(a) Compute the total energy E of all electrons in a system of a given volume.

(b) Compute the Fermi energy E_F as a function of the concentration n.

(c) Differentiating the energy E with respect to the volume V, obtain the pressure P. Express it through the concentration n and the Fermi energy E_F.

(d) Recall the definition of compressibility. Using the formula for pressure you obtained, compute the compressibility of copper.

2. In a white dwarf (a star consisting of fully ionised atoms and electrons), the high density of matter leads to extremely large Fermi energies of the electron gas, $E_F \gg m_e c^2$, where m_e is the electron mass, which also exceeds significantly the temperature T. Obtain the equation of state of this electron gas in the variables V, P.

3. When a certain metal is compressed isotropically, its Fermi energy increases by 0.1%. Compute the relative change of the Debye frequency as a result of this compression process, assuming that the speed of sound is constant.

4. Estimate the temperature of the Sun, taking into account that the angular size of the Sun in the sky is $\alpha \approx 0.01$ and the temperature of the Earth's surface is $T_E \approx 300K$.

Hint: Assume for simplicity that the Earth absorbs all incoming radiation (in reality it absorbs about 70%).

5. A vacancy is a defect in a crystal which is created by removing an atom from a node of the crystalline lattice. Clearly, in any crystal in the equilibrium state at any finite temperature there exists a finite concentration of vacancies. When a crystal is cooled down sufficiently quickly, the number of vacancies corresponding to the equilibrium at high temperature does not change, i.e. vacancies remain "frozen". After that the system relaxes very slowly to a new equilibrium state; this process is called the annealing of vacancies. Compute the temperature change during an adiabatic vacancy annealing in a sample of aluminium which was cooled from $T_2 = 660°C$ to the room temperature T_1. Assume that the heat capacity of aluminium is described by the classical theory ($T \gg \theta_D$, with θ_D being the Debye temperature). The energy of creating a vacancy in aluminium is $E = 0.75eV$.