Homework 2 answers

(1) See Landau-Lifshitz, V. 5, § 39

(2) a) $\nu(\varepsilon) = \frac{m}{2\pi\hbar^2}$

b) $\langle \varepsilon \rangle = T$

c) $\langle \varepsilon^2 \rangle = \frac{\int_0^\infty e^{-\frac{\varepsilon}{T}} \varepsilon^2 d\varepsilon}{\int_0^\infty e^{-\frac{\varepsilon}{T}} d\varepsilon} = 2T^2$

(3) $\langle \hat{S}_z \rangle = \frac{2 \sinh\left(\frac{B}{T}\right)}{1 + 2\cosh\left(\frac{B}{T}\right)}$

(4) $\langle \varepsilon \rangle = \frac{\hbar\omega}{e^{\frac{\hbar\omega}{T}} - 1} + \frac{\hbar\omega}{2}$

(5) $\langle U \rangle = T$

Boltzmann distribution: $n = n_0 e^{-\frac{mgz}{T}}$

$\left(n_0 = \frac{P}{T}\right)$

Total number of particles:

$N = \int_0^\infty n_0 e^{-\frac{mgz}{T}} S\,dz = n_0 S \frac{T}{mg} = \frac{PS}{mg}$

$$C = N\left(\frac{\partial\langle\varepsilon\rangle}{\partial T} + \frac{\partial\langle U\rangle}{\partial T}\right) = N\left(\frac{5}{2} + 1\right) = \frac{7}{2}N$$

Assuming diatomic molecules

Note: $C = C_P$ (regardless of any assumptions about molecules in the air)