Homework 3 solutions

1. Thermal expansion coefficient

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P = \frac{1}{V}\frac{\left(\frac{\partial V}{\partial S}\right)_P}{\left(\frac{\partial T}{\partial S}\right)_P} = \frac{T}{V\,C_p}\left(\frac{\partial V}{\partial S}\right)_P$$

$\rightarrow$ The signs of $\left(\frac{\partial S}{\partial V}\right)_P$ and α coincide

2. $mc \ln \frac{T_2}{T_1}$

3. Similarly to 1

4. $$\Delta S = \frac{4}{3}\frac{E}{T}(V_2 - V_1)$$

$$W = \frac{4}{3} a T_1^3 (T_1 - T_2)(V_2 - V_1)$$

Hint: take into account that the internal energy of the radiation depends on its volume

5. a) $dE = T\,dS + f\,dl$

$-f$ plays the role of pressure

l is the generalised volume

$$\left(\frac{\partial E}{\partial l}\right)_T = T\underbrace{\left(\frac{\partial S}{\partial l}\right)_T}_{\overset{\text{Maxwell relation}}{=} -\left(\frac{\partial f}{\partial T}\right)_l} + f = -f + f = 0$$

$\Longrightarrow$ The internal energy is length-independent

b) $\left(\frac{\partial S}{\partial l}\right)_T = -\left(\frac{\partial f}{\partial T}\right)_l < 0$