Condensed Matter 232 WQ 2019

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Solve by 18 January 2019

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1. Compute the Fermi energy, Fermi momentum and Fermi velocity for a metal with the quadratic quasiparticle dispersion $\varepsilon_{\mathbf{k}} = \frac{k^2}{2m^*}$, where $m^* = 0.8m_e$ is the effective mass of the quasiparticles. The concentration of electrons is $n = 10^{23}cm^3$.

2. Quasiparticles in 1D wires.

Modern semiconductor technology allows for fabrication of quantum wires based on heterostructures AlGaAs-GaAs-AlGaAs, 1D systems with a quadratic electron dispersion with the mass $m^* = 0.07m_e$. In the transverse direction the electrons are confined to a potential well with one bound level whose depth is $E_0 = 50meV$, as shown in Fig. 1. What is the maximum achievable density of electrons (per length) in such a system?

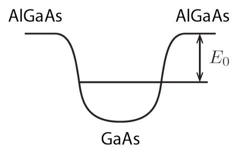

Figure 1: Potential as a function transverse coordinate in a quantum wire.

3. Heat capacity of graphene.

In graphene, a two-dimensional layer of carbon atoms, the quasiparticle dispersion is given by $\varepsilon_{\mathbf{k}} = v\mathbf{k} \cdot \hat{\boldsymbol{\sigma}}$, where v is a constant (called "Fermi velocity" in graphene), $\mathbf{k}$ is the quasimomentum measured from some point (node) in the Brillouin zone and $\hat{\boldsymbol{\sigma}}$ is a degree of freedom, called "pseudospin", equivalent to spin-1/2.

(a) Find the density of quasiparticle states in graphene.

(b) In the absence of external gate electrodes and dopant impurities (undoped graphene), the chemical potential matches the energy of the quasiparticles at $\mathbf{k} = 0$. Compute the heat capacity of an undoped graphene sample.