Distribution function. Kinetic equation.

Consider quasiclassical motion of electrons: the system is exposed to perturbations with typical length scales $R \gg \lambda_F$ and slow in time, $E_F \gg \omega$.

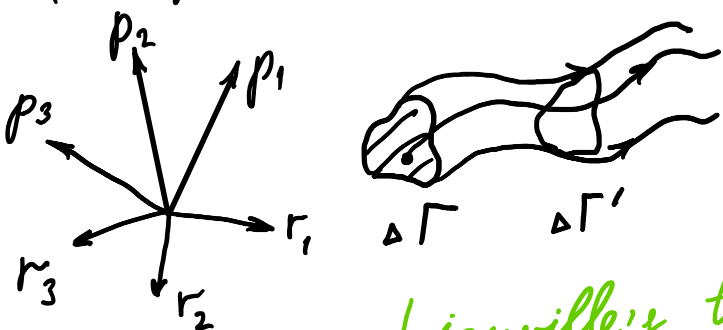
It is possible to talk about coordinate R and momentum p simultaneously

$$f(\vec{r}, \vec{p}, t)$$

$\frac{\Delta p \Delta r}{(2\pi\hbar)^3}$ – the number of states in the element $\Delta p \Delta r$ of phase space (in 3D)

So, $f(\vec{r}, \vec{p}, t)$ is the density of quasiparticles

Quasiparticles move almost classically



Liouville's theorem: $\Delta\Gamma = \Delta\Gamma'$ $\left(\frac{df}{dt} = 0\right)$

▷ Proof

Classical equations of motion: $\begin{cases} \dot{\vec{r}} = \frac{\partial H}{\partial \vec{p}} \\ \dot{\vec{p}} = -\frac{\partial H}{\partial \vec{r}} \end{cases}$

Continuity equation:

Continuity eq.:

$$\frac{\partial f}{\partial t} + \frac{\partial (f\dot{\vec{r}})}{\partial \vec{r}} + \frac{\partial (f\dot{\vec{p}})}{\partial \vec{p}} = 0 \quad \left(\frac{\partial \rho}{\partial t} + \operatorname{div} \vec{j} = 0\right)$$

$$f\frac{\partial \dot{\vec{r}}}{\partial \vec{r}} + f\frac{\partial \dot{\vec{p}}}{\partial \vec{p}} = f\frac{\partial^2 H}{\partial \vec{p}\,\partial \vec{r}} - \frac{\partial^2 H}{\partial \vec{r}\,\partial \vec{p}} f = 0$$

$$\longrightarrow \quad \frac{df}{dt} \equiv \frac{\partial f}{\partial t} + \frac{\partial f}{\partial \vec{r}}\dot{\vec{r}} + \frac{\partial f}{\partial \vec{p}}\dot{\vec{p}} \quad \triangleleft$$

Kinetic equation

$$\frac{df}{dt} = \underbrace{St\ f}_{\text{Collision integral}} \quad (= I(f))$$

Collision integral (random disorder, interactions, e-ph interactions, ...)

In a conducting system in an external electric field $\vec{E}$, $\dot{\vec{p}} = e\vec{E}$, $\dot{\vec{r}} \equiv \vec{v}_{\vec{p}}$

$$\boxed{\frac{\partial f}{\partial t} + \vec{v}_{\vec{p}}\frac{\partial f}{\partial \vec{p}} + e\vec{E}\frac{\partial f}{\partial \vec{r}} = St\ f}$$

Boltzmann kinetic equation

Collision integral in the τ-approximation:

$$St\ f = -\frac{f - f_0}{\tau}$$

(τ – „relaxation" or „scattering" time)